#

Algebra 2/Trig
Unit 1 Matrices Notes Packet

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Matrices

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A matrix is a rectangular arrangement of numbers in rows and columns. Rows run _________________ and columns run______________________.

The ________________ of the matrix below are 2 x 3 (read 2 by 3). We talk about matrices by naming the row first and the column second. The numbers in a matrix are called its _____________.

$$A = \underbrace{\begin{bmatrix} 6 & 2 & -1 \\ -2 & 0 & \color{red}{5} \end{bmatrix}}_{\textbf{3 columns}} \left.\vphantom{\begin{bmatrix} 6 \\ -2 \end{bmatrix}}\right\} \textbf{2 rows}$$

Two matrices are ____________ if their dimensions are the same and the entries in corresponding positions are equal.

Some matrices have special names because of what they look like.

_________________________: only has 1 row.

_________________________: only has 1 column.

_________________________: has the same number of rows and columns.

_________________________: contains all zeros

To add or subtract matrices, you simply add or subtract corresponding _______________.

***NOTE: You can add or subtract matrices only if they have the same __________________.**

E1. $\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ -2 & 1 \\ 3 & -3 \end{bmatrix}$

P1. $\begin{bmatrix} -7 & 5 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 1 \\ -4 & 2 \end{bmatrix}$

E2. $\begin{bmatrix} -2 & 0 & 4 \\ 3 & -10 & 12 \\ 3 & -2 & -2 \end{bmatrix} + \begin{bmatrix} -4 & 6 & 0 \\ -15 & 2 & -4 \\ 6 & 7 & 1 \end{bmatrix}$

P2. $\begin{bmatrix} 7 & -8 & 10 \\ -3 & 6 & 1 \\ 5 & -4 & 0 \end{bmatrix} + \begin{bmatrix} 11 & -4 & 10 \\ 2 & -6 & -4 \\ -9 & 12 & -3 \end{bmatrix}$

In matrix algebra, a real number is often called a ___________________.

To multiply a matrix by a ________________, you must multiply each entry in the matrix by the ____________. This process is called _____________________ ____________________.

E3. $4\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -2 \end{bmatrix}$

P3. $-3\begin{bmatrix} 4 & 3 \\ -6 & 8 \\ 0 & -7 \end{bmatrix}$

E4.

$$2\begin{bmatrix} -3 & 10 & 2 \\ -10 & 8 & -6 \\ 0 & 1 & 0 \end{bmatrix}$$

P4.

$$-3\begin{bmatrix} 4 & -5 & 20 \\ -7 & -13 & 7 \\ -2 & -5 & 10 \end{bmatrix}$$

E5. Solve the matrix equation for x and y.

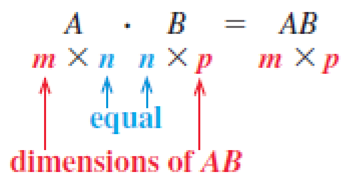
$$4\left(\begin{bmatrix} 8 & 0 \\ -1 & 2y \end{bmatrix} + \begin{bmatrix} 4 & -2x \\ 1 & 6 \end{bmatrix}\right) = \begin{bmatrix} 48 & -48 \\ 0 & 8 \end{bmatrix}$$

P5. Solve the matrix equation for x and y.

$$3\left(\begin{bmatrix} 10 & 2 \\ 5 & 4y \end{bmatrix} - \begin{bmatrix} x & 5 \\ -1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 & -9 \\ 18 & 21 \end{bmatrix}$$

4.2 Multiplying Matrices (I,E)

The ____________________ of two matrices A and B is defined provided that the number of ______________ in A is equal to the number of _________ in B.



E1. State whether the product AB is defined. If so state the dimensions of AB.

a. A: 2 x 3, B: 3 x 4

b. A: 3 x 2, B: 3 x 4

P1. State whether the product AB is defined. If so state the dimensions of AB.

a. A: 2 x 4, B: 4 x 3

b. A: 1 x 4, B: 1 x 4

E2: If A = $\begin{bmatrix} -3 & 2 & 5 \\ 7 & 1 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} -8 & 2 \\ 0 & -3 \end{bmatrix}$, find each product

a. AB

b. BA

P2: If A = $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix}$, find each product

a. AB

b. BA

E3. If A = $\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -4 \\ 2 & 1 \end{bmatrix}$, find each product

a. AB

b. BA

P3. If A = $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & -3 \\ -2 & 5 \end{bmatrix}$, find each product

a. AB

b. BA

E4. Solve for x and y.

$$\begin{bmatrix} 1 & 3 & -2 \\ -1 & 3 & 2 \\ 2 & -4 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ x \end{bmatrix} = \begin{bmatrix} -8 \\ 2 \\ y \end{bmatrix}$$

P4. Solve for x and y.

$$\begin{bmatrix} 0 & 8 \\ -11 & -9 \end{bmatrix}\begin{bmatrix} 3 & 9 \\ 18 & x \end{bmatrix} = \begin{bmatrix} 144 & 40 \\ y & -144 \end{bmatrix}$$

Solving Systems of Linear Equations

1. Graphing
2. Substitution
3. Elimination
4. *Cramer's Rule*-Cramer's Rule is a method of solving systems of equations using a special number known as the determinant of the matrix.

Determinant:
Every square matrix has a real number associated with it known as a determinant. The determinant of matrix A is denoted by det A or by $|A|$.

E1. $\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix}$

P1. $\begin{vmatrix} 7 & 2 \\ 2 & 3 \end{vmatrix}$

E2. Evaluate the determinant of the matrix. $\begin{bmatrix} 2 & -1 & 3 \\ -2 & 0 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

P2. Evaluate the determinant of the matrix. $\begin{bmatrix} 4 & 3 & 1 \\ 5 & -7 & 0 \\ 1 & -2 & 2 \end{bmatrix}$

Cramer's Rule:
You can use determinants to solve a system of linear equations. The method, called Cramer's Rule and named after the Swiss mathematician Gabriel Cramer (1704-1752), uses the coefficient matrix of the linear system.

2 X 2
Let A be the coefficient matrix of the linear system:

$$ax + by = e$$
$$cx + dy = f$$

If det $A \neq 0$, then the system has exactly one solution. The solution is:

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \quad and \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$$

Use Cramer's Rule to solve the system

E3. $8x + 5y = 2$
$2x - 4y = -10$

P3. $2x + y = 1$
$3x - 2y = -23$

3 X 3
Let A be the coefficient matrix of this linear system:

$$ax + by + cz = j$$
$$dx + ey + fz = k$$
$$gx + hy + iz = l$$

If det $A \neq 0$, then the system has exactly one solution. The solution is:

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A}, \quad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A}, \quad and \quad z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}$$

Use Cramer's Rule to solve the system

E4. $x + 2y - 3z = -2$
$x - y + z = -1$
$3x + 4y - 4z = 4$

P4. $x + 3y - z = 1$
$-2x - 6y + z = -3$
$3x + 5y - 2z = 4$

Using the TI Graphing Calculator with matrices to solve a system of linear equations

$$8x + 5y = 2$$
$$2x - 4y = -10$$

1. Input the coefficient matrix by following these steps (Matrix A)

$$\begin{bmatrix} 8 & 5 \\ 2 & -4 \end{bmatrix}$$

- Matrix (2nd and x^{-1})
- Edit
- 1

2. Repeat Step 1: Input the x matrix (Matrix B)

$$\begin{bmatrix} 2 & 5 \\ -10 & -4 \end{bmatrix}$$

3. Repeat Step 1: Input the y matrix (Matrix C)

$$\begin{bmatrix} 8 & 2 \\ 2 & -10 \end{bmatrix}$$

4. Use the calculator to determine the x – value of the solution

- Matrix
- Math
- det
- Matrix
- B
-)
- ÷
- Matrix
- Math
- det
- Matrix
- A
- Enter

5. Use the calculator to determine the y – value of the solution

- Matrix
- Math
- det
- Matrix
- C
-)
- ÷
- Matrix
- Math
- det
- Matrix
- A
- Enter

Compare your answer with E3

The number 1 is the multiplicative identity for real numbers because $1 * a = a$ and $a * 1 = a$. For matrices, the $n\ x\ n$ ___________________ _________________ is the matrix that has 1's on the main diagonal and 0's elsewhere.

2 × 2 IDENTITY MATRIX

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3 × 3 IDENTITY MATRIX

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If A is any $n\ x\ n$ matrix and *I* is the $n\ x\ n$ identity matrix, then $IA = A\ and\ AI = A$.

Two $n\ x\ n$ matrices are _______________ of each other if their product, in BOTH orders, is the $n\ x\ n$ identity matrix.

For example, if A = $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ when we find AB it equals $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ which is the identity matrix (*I).*

The symbol used for the inverse of A is A^{-1}.

THE INVERSE OF A 2 X 2 MATRIX

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ provided } ad - cb \neq 0.$$

Some matrices do not have an inverse. You can check for this by evaluating the determinant of A. If det A = 0, then there is no inverse.

E1. Find the inverse of A = $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

Check:

P1. Find the inverse of A = $\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$

Check:

To solve a matrix equation:

(1) Find the inverse of A
(2) Multiply both sides of the equation by the inverse (A^{-1}) ON THE LEFT.
(3) Check – by multiplying A and X to see if you get B

E2. Solve the matrix equation AX = B for the 2 x 2 matrix X

$$\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} X = \begin{bmatrix} 8 & -5 \\ -6 & 3 \end{bmatrix}$$

P2. Solve the matrix equation AX = B for the 2 x 2 matrix X

$$\begin{bmatrix} -3 & 4 \\ 5 & -7 \end{bmatrix} X = \begin{bmatrix} 3 & 8 \\ 2 & -2 \end{bmatrix}$$

The inverse of a 3 x 3 matrix should be done on the graphing calculator

E3. Find the inverse of A.

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$$

P3. Find the inverse of A.

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & -1 & -2 \\ 3 & 1 & 1 \end{bmatrix}$$

A *cryptogram* is a message written according to a secret code. (The Greek word *kruptos* means *hidden* and the Greek word *gramma* means *letter*.) The following technique uses matrices to encode and decode messages.

First assign a number to each letter in the alphabet with 0 assigned to a blank space.

__ = 0	E = 5	J = 10	O = 15	T = 20	Y = 25
A = 1	F = 6	K = 11	P = 16	U = 21	Z = 26
B = 2	G = 7	L = 12	Q = 17	V = 22	
C = 3	H = 8	M = 13	R = 18	W = 23	
D = 4	I = 9	N = 14	S = 19	X = 24	

Then convert the message to numbers partitioned into 1×2 *uncoded row matrices.*

To *encode* a message, choose a 2×2 matrix A that has an inverse and multiply the uncoded row matrices by A *on the right* to obtain *coded row matrices.*

E4. Use A = $\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$ to encode the message GET_HELP

P4. Use A = $\begin{bmatrix} 1 & 4 \\ 2 & 6 \end{bmatrix}$ to encode the message THANK_YOU

DECODING USING MATRICES Decoding the cryptogram created in Example 5 would be difficult for people who do not know the matrix A. When larger coding matrices are used, decoding is even more difficult. But for an authorized receiver who knows the matrix A, decoding is simple. The receiver only needs to multiply the coded row matrices by A^{-1} *on the right* to retrieve the uncoded row matrices.

E5. Use the inverse of A = $\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$ to decode this message:

−4, 3, −23, 12, −26, 13, 15, −5, 31, −5, −38, 19, −21, 12, 20, 0, 75, −25

P5. Use the inverse of A = $\begin{bmatrix} -1 & 1 \\ 2 & -3 \end{bmatrix}$ to decode this message:

-4, -1. -12, 7, -18, 18, 11, -20, -12, 7, 42, -63, -16, 16

Use AX = B where A is the matrix of ______________, X is the matrix of _________________ and B is the matrix of _________________.

$AX = B$	Original matrix equation
$A^{-1}AX = A^{-1}B$	Multiply both sides by the inverse of A
$IX = A^{-1}B$	$A^{-1}A$ is the identity matrix *I*
$X = A^{-1}B$	Use this equation to solve for the variables

E1. Use matrices to solve the linear system. $-3x + 4y = 5$ and $2x - y = -10$

P1. Use matrices to solve the linear system. $-2x - 5y = -19$ and $3x + 2y = 1$

E2. Use matrices and a graphing calculator to solve the linear system.

$2x + 3y + z = -1$

$3x + 3y + z = 1$

$2x + 4y + z = -2$

P2. Use matrices and a graphing calculator to solve the linear system.

$2x + 3y - z = 14$

$4x + 5y + 2z = 34$

$-x + 3y - 4z = 20$

Warm-ups

Use the provided spaces to complete any warm-up problem or activity

Date:	Date:
Date:	Date:
Date:	Date:
Date:	Date:
Date:	Date:

Warm-ups

Use the provided spaces to complete any warm-up problem or activity

Date:	Date:
Date:	Date:
Date:	Date:
Date:	Date:
Date:	Date:

Warm-ups

Use the provided spaces to complete any warm-up problem or activity

Date:	Date:
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